QP CODE: 21100166

21100166

Reg No : .....

Name : .....

B.Sc DEGREE (CBCS) EXAMINATION, FEBRUARY 2021

Fifth Semester

Core Course - MM5CRT01 - MATHEMATICAL ANALYSIS

B.Sc Mathematics Model I, B.Sc Mathematics Model II Computer Science, B.Sc Computer Applications Model III Triple Main

2017 Admission Onwards

C9A38860

Time: 3 Hours Max. Marks : 80

Part A

*Answer any **ten** questions.*

Each question carries 2 marks.

1. State ordering property of real numbers? Is the set of all rational numbers ordered? Justify.
2. Find all $x \in R$ such that $|x-1| > |x+1|$?
3. Does $f(x) \leq g(x)\ \forall x \in D$ imply that $Sup\ f(D) \leq Inf\ g(D)$? Give proper reasoning?
4. Define Nested intervals? Is the interval $I_n = \left(-\frac{1}{n}, \frac{1}{n}\right), n \in N$ nested?
5. Prove that the sequence $(x_n) = ((-1)^n)$ does not converge.
6. Prove that a convergent sequence of real numbers is bounded.
7. Find $lim(\frac{n+1}{n\sqrt{n}})$.
8. Using Monotone Convergence Theorem, prove that $lim(\frac{1}{\sqrt{n}}) = 0$.
9. Use the recurrance relation of nth term of a sequence that converges to $\sqrt{a}$ to find the value of $\sqrt{5}$ correct to 4 decimal places.
10. State Abel's Lemma.
11. Test the convergence of $\sum_1^\infty \frac{(-1)^{n+1}}{(n^2+1)}$
12. Show that $\lim_{x\to\infty} x^n = \infty$ for $n \in \mathcal{N}$.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Prove that If A, B are bounded sets then $Sup\,(A+B) = Sup\,A + Sup\,B$ where $A+B = \{a+b : a \in A, b \in B\}$

14. Prove that $x \in [0,1]$ then the binary representation of x forms a sequnce consisting only $0, 1$?

15. If $c > 0$, prove that $\lim (c^{1/n}) = 1$.

16. Prove that (sin n) is divergent.

17. Let (x_n) and (y_n) be two sequences of real numbers and suppose that $x_n \leq y_n$ for all n. Prove that
    (a) if $\lim x_n = +\infty$ then $\lim y_n = +\infty$.
    (b) if $\lim y_n = -\infty$ then $\lim x_n = -\infty$.

18. Prove that the geometric series $\sum_{n=0}^{n=\infty} r^n$ converges if and only if $|r| < 1$.

19. If (a_n) is a decreasing sequence of strictly positive numbers and if $\sum a_n$ is convergent, show that $\lim na_n = 0$.

20. Show that $\lim_{x\to 0} sgn(x)$ does not exist.

21. Let $A \subseteq \mathscr{R}$, $f, g : A \to \mathscr{R}$, $c \in \mathscr{R}$ be a cluster point of A. If $f(x) \leq g(x)$ for all $x \in A, x \neq c$.Then prove the following
    (a) If $\lim_{x\to c} f = \infty$, then $\lim_{x\to c} g = \infty$.
    (b) If $\lim_{x\to c} g = -\infty$, then $\lim_{x\to c} f = -\infty$.

(6×5=30)

Part C

Answer any two questions.

Each question carries 15 marks.

22. Prove the denumerability of the following sets
    (a.) The set of all rational numbers Q
    (b) The set $N \times N$, where N is the set of all natural numbers

23. (a) State and prove Cauchy Convergence Criterion.
    (b) Let $X = (x_n)$ be the sequence defined as $x_1 = 1$, $x_2 = 2$ and $x_n = \frac{x_{n-2}+x_{n-1}}{2}$ for $n > 2$. Prove that $\lim X = \frac{5}{3}$.

24. State and prove Raabe's test. Use this test to study the convergence of $\sum_1^\infty (\frac{n}{(n^2-1)})$.

25. (a) Let $A \subseteq \mathscr{R}$, $f : A \to \mathscr{R}$ and let $c \in \mathscr{R}$ be a cluster point of A. If $a \leq f(x) \leq b$ for all $x \in A$, $x \neq c$, and if $\lim_{x\to c} f$ exists,Then prove that $a \leq \lim_{x\to c} f \leq b$.
    (b) Check whether the following limits exist or not. Give explanations
    (1) $\lim_{x\to 0} cos(\frac{1}{x})$ (2) $\lim_{x\to 0} xcos(\frac{1}{x})$

(2×15=30)